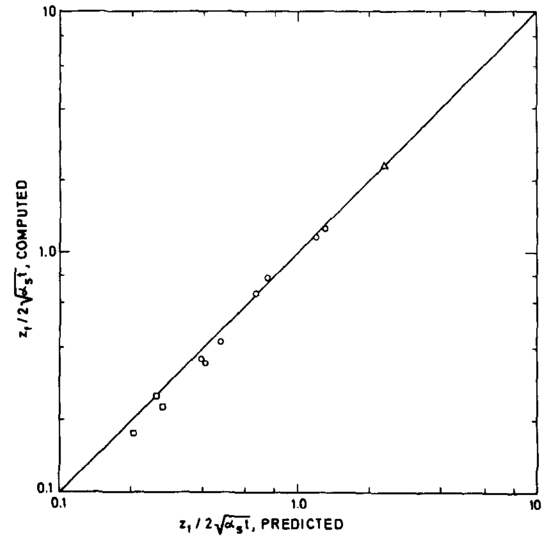

FIG. 3. Freezing-front location along the plane of symmetry in a square corner.

Symbols / Authors	$\frac{k_l(T_0-T_f)}{k_s(T_f-T_w)}$	$\frac{\alpha_s}{\alpha_l}$	$\frac{L}{C_s(T_f-T_w)}$
□ Jiji *et al.* [9]	1.35	9.2	33.9
	0.553	9.2	22.4
	0.250	9.2	19.6
△ Lazaridis [10]	0.257	1.111	0.50
○ Rathjen and Jiji [12]	0.5, 2.00	1.0	0.1–10

CONCLUSIONS

Solutions for pure conduction with the effective diffusivities defined by equations (10) and (12) were found to agree closely with exact solutions for the heat flux density and the freezing front location, respectively. Similar accuracy is to be expected for the heat flux density and phase-front location in any geometry and with any boundary conditions, except that the freezing-front location cannot be calculated from equation (8) if $T_0 - T_f$. Corresponding approximations for the cases of convection in the liquid phase, a freezing range, moisture migration in wet soil, etc., can readily be formulated. The detailed temperature fields can also be calculated using these approximations, with equation (10) expected to give a better representation for the region near the surface and equation (12) near the freezing front.

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PERTURBATION SOLUTION FOR CONVECTIVE FIN WITH INTERNAL HEAT GENERATION AND TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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NOMENCLATURE

Bi, Biot number, $= hw/2k_a$;
E, fin effectiveness, $= Q/hw(T_b - T_a)$;
G, generation number, $= qw/2h(T_b - T_a)$;
h, heat-transfer coefficient;
k, thermal conductivity;
L, fin length;
N, fin parameter, $= \left(\frac{2h}{k_a w}\right)^{1/2} L$;
q, volumetric rate of heat generation;
Q, heat-transfer rate;
T, temperature;
w, fin thickness;
x, axial distance measured from fin tip;
X, dimensionless axial distance, $= x/L$.

Greek symbols

β, slope of thermal conductivity–temperature curve divided by intercept k_a;
θ, dimensionless temperature;
ε, thermal conductivity parameter; $(k_b - k_a)/k_a = \beta(T_b - T_a)$.

Subscripts

a, environment;
b, fin base.

INTRODUCTION

IN FIN literature one finds several papers focussing attention on the effect of internal heat generation on the performance of convective fins. For example, Minkler and Rouleau [1] studied rectangular and triangular fins with uniform internal

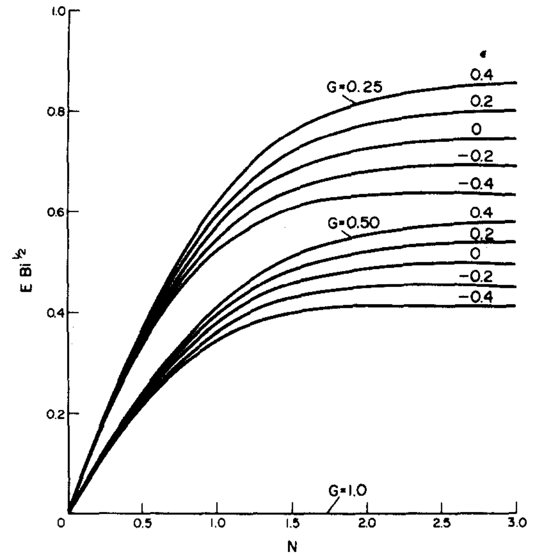

FIG. 1. Effect of temperature dependent thermal conductivity on heat-transfer rate for a rectangular fin with uniform internal heat generation.

heat generation but constant thermal conductivity and heat-transfer coefficient. A more general analysis for arbitrary fin profile with coordinate dependent heat generation, heat-transfer coefficient and thermal conductivity was presented by Melese and Wilkins [2]. Invoking the assumption of constant thermal conductivity Ahmadi and Razani [3] derived the optimal shape of a convective fin with coordinate dependent heat generation.

The present work considers a uniformly thick rectangular convective fin with constant internal heat generation but linear thermal conductivity–temperature variation. This nonlinear problem can be solved numerically to predict the temperature distribution and heat-transfer rate. However, the problem of optimising fin dimensions still remains difficult. Here, as an alternative, an approximate perturbation solution is developed in which the constant thermal conductivity solution forms the zero order solution. Despite the approximation, the solution gives accurate results covering the range of parameters met in practice. Besides providing accurate prediction for temperature and heat-transfer rate, the solution permits (in a straight-forward manner) the optimisation of fin dimensions taking into consideration the effect of variable thermal conductivity. The present paper constitutes an extension to the author's previous work [4, 5] which dealt with a nongenerating convective fin having temperature-dependent thermal conductivity.

ANALYSIS

Consider a rectangular fin of length L and thickness w generating heat at the rate of q per unit volume. Both faces of the fin convect heat to the environment at temperature T_a and with constant heat-transfer coefficient h. The usual boundary conditions of constant base temperature T_b and insulated tip are assumed. Further, it is assumed that the thermal conductivity varies linearly with temperature and the relationship is expressed as

$$k = k_a[1+\beta(T-T_a)]. \tag{1}$$

Placing the origin at the tip, the one-dimensional fin equation together with aforesaid boundary conditions can be written in dimensionless form as

$$\frac{\mathrm{d}}{\mathrm{d}X}\left[(1+\varepsilon\theta)\frac{\mathrm{d}\theta}{\mathrm{d}X}\right]-N^2(\theta-G)=0 \tag{2}$$

$$X=0,\ \frac{\mathrm{d}\theta}{\mathrm{d}X}=0;\quad X=1,\ \theta=1 \tag{3}$$

where

$$\begin{gathered} X=x/L,\ \theta=(T-T_a)/(T_b-T_a),\ N^2=2hL^2/k_a w, \\ G=qw/2h(T_b-T_a)=\text{generation number} \\ \varepsilon=(k_b-k_a)/k_a=\beta(T_b-T_a). \end{gathered} \tag{4}$$

Since for most fin materials and operational temperatures, $\varepsilon \ll 1$ is valid, an asymptotic expansion of the form

$$\sum_{n=0}^{\infty}\varepsilon^n\theta_n \tag{5}$$

is appropriate. Substituting (5) into (2) and (3) and equating coefficients of like powers of ε, a set of linear boundary value problems for θ_0, $\theta_1,\ldots,$ etc. are generated which can be solved successively. Specifically, solutions for θ_0, θ_1 and θ_2 were obtained but since the contribution of the second order term was found to be small, its presentation here is omitted to conserve space. Without giving the intermediate mathematical details, the solution correct to $O(\varepsilon)$ is

$$\begin{aligned} \theta = {} & G+(1-G)\operatorname{sech}N\cosh NX \\ & +\varepsilon\{[\tfrac{1}{3}(1-G)^2\operatorname{sech}^3N\cosh 2N \\ & +\tfrac{1}{2}G(1-G)N\operatorname{sech}N\tanh N]\cosh NX \\ & -\tfrac{1}{3}(1-G)^2\operatorname{sech}^2N\cosh 2NX \\ & -\tfrac{1}{2}N\operatorname{sech}NG(1-G)X\sinh NX\}+O(\varepsilon^2). \end{aligned} \tag{6}$$

The accuracy of the perturbation solution was checked by solving equations (2) and (3) numerically on a HP2100S digital computer. Sixty solutions were obtained using the parametric values of $N = 1.0, 1.5, 2.0, 2.5, 3.0$; $\varepsilon = \pm 0.2, \pm 0.4$; $G = 0.1, 0.25, 0.50$ which cover a wide range of fin applications. The maximum error between the perturbation and the numerical solutions was found to be about 2%. Detailed display of these results is omitted in favour of more useful design information on heat-transfer rate and optimisation which follows.

Using equation (6) the rate of heat-transfer Q patterned on the form given in [6] follows as

$$\begin{aligned} \frac{QL}{k_a w(T_b-T_a)N} &= EBi^{1/2} \\ &= (1-G)\tanh N+\varepsilon[\tfrac{1}{3}(1-G)^2\tanh^3N \\ &\quad +\tfrac{1}{2}G(1-G)(\tanh N-N\operatorname{sech}^2N)]+O(\varepsilon^2) \end{aligned} \tag{7}$$

where

$$\begin{aligned} E &= Q/hw(T_b-T_a) = \text{fin effectiveness} \\ Bi &= hw/2k_a = \text{Biot number.} \end{aligned} \tag{8}$$

A design chart based on equation (7) appears as Fig. 1 where $EBi^{1/2}$ is plotted against N for $G = 0.25$ and 0.50 with ε as the curve parameter. The values of G chosen here are typically encountered in practice. For example, [1] quotes values of G of 0.20–0.25 for finned nuclear reactor fuel elements. Figure 1 shows that as N increases, the effect of variable thermal conductivity on parameter $EBi^{1/2}$ becomes more pronounced. However, the effect is seen to diminish as G increases. For $G = 1$, $EBi^{1/2}$ is always zero and represents the situation when there is no heat removal from the primary surface and only the internally generated heat is convected out from the fin surface.

The criterion for optimum dimensions is that for a fixed profile area, wL, the heat-transfer rate should be maximum. Thus, from equation (7) the condition $\mathrm{d}Q/\mathrm{d}w = 0$ gives the following transcendental equation for optimum N

$$\begin{aligned} & (1-3G)\sinh 2N-6(1-G)N \\ & +\varepsilon[\tfrac{1}{3}(1-6G+5G^2)\sinh 2N\tanh^2N \\ & -6(1-G)^2N\tanh^2N+\tfrac{1}{2}G(3-5G)\sinh 2N \\ & -6G(1-G)N^2\tanh N-G(3-5G)N]=0. \end{aligned} \tag{9}$$

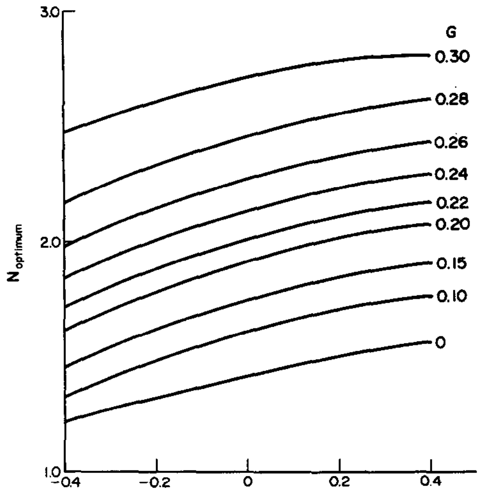

FIG. 2. Optimisation chart for rectangular fin with uniform internal heat generation and temperature dependent thermal conductivity.

The optimisation data is presented in Fig. 2 wherein N_{opt} obtained by solving equation (9) is plotted against ε for a range of values of G. With the aid of Fig. 2, the optimum dimensioned fin for a specified heat generation can be readily designed allowing for thermal conductivity variation.

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INVESTIGATION OF TEMPERATURE FIELDS IN HEAT EXCHANGERS OF POROUS CYLINDRICAL BOARDS

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NOMENCLATURE

A, B, dimensionless parameters;
C_{l2}, C_{l3}, constants;
C_p, specific heat of the fluid;
J_0, J_1, Bessel functions;
K_l, coefficients of evolution of temperature field;
$\dot{Q}$, total heat flow;
$\dot{Q}_0$, ideal heat flow;
R, $= r/r_0$, dimensionless radius, co-ordinate;
r, radius, co-ordinate;
r_0, radius of porous board;
T_f, temperature of fluid;
T_{f0}, temperature of incoming fluid;
T_s, temperature of porous material;
T_{s0}, temperature of circumference of porous board;
w, specific mass throughflow;
Z, $= z/r_0$, dimensionless co-ordinate;
z, co-ordinate;
Z_0, $= z_0/r_0$, dimensionless height of porous board;
z_0, height of porous board.

Greek symbols

α, coefficient of heat transfer;
α_l, zero points of function J_0;
β, specific area of heat transfer;
γ, $= \lambda_z/\lambda_r$, rate of orthotropy;
Θ_s, dimensionless temperature of porous material;
Θ_f, dimensionless temperature of fluid;
λ_{il}, roots of characteristic polynome;
λ_z, λ_r, thermal conductivities in axial and in radial directions;
ϕ_l, component of temperature field;
φ, co-ordinate.

INTRODUCTION

THE HEAT exchangers of porous materials are of importance in number of applications, for example in an effective utilization of enthalpy of outgoing gaseous helium in throughflow cryostats, in refrigerators making use of dissolution of ^3He in ^4He [1].

This article presents a solution of stabilized temperature fields in a orthotropic porous material of cylindrical shape, thermally connected by its circumference to a body with temperature T_{s0}. It can be higher or lower than the temperature of incoming cooling (warming) medium T_{f0}, flowing only in axial direction through the porous material (Fig. 1).

The solution is found on the following assumptions:

(a) The geometrical and physical parameters are axially symmetrical.

(b) We regard the porous substance as a continuous and homogeneous environment (i.e. neglecting the microstructure).

(c) The heat is brought in solely by the outer circumference of the porous board, and removed by a transfer into the fluid, or conversely.